**Understanding Underdamped and Overdamped Oscillations**

**Introduction**  
In my exploration of oscillatory motion, I focused on underdamped and overdamped systems. Both scenarios highlight the effect of damping on the motion of a system, which is crucial in fields like engineering and physics. By analyzing these systems, I aimed to derive the equations of motion and understand how energy dissipates over time.

**Underdamped Oscillation**

For the underdamped case, I started with a system displaced by 25 cm with no initial velocity. The damping was low, allowing oscillations to persist but gradually decrease in amplitude. I used Guy and Collie’s equation, which simplifies the solution under specific conditions:

1. **Zero Initial Velocity:** Ensures the cosine term dominates.
2. **Small Damping Coefficient (γ):** The damping factor must be significantly less than the natural frequency (ω₀).

The motion was governed by the equation:

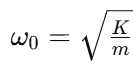


Where:

A: Initial amplitude (25 cm or 0.25 m).

 : Damping coefficient.

 : Damped angular frequency.

 : Natural angular frequency.

**Steps for Derivation:**

1. **Calculate Amplitude (A):**



1. **Determine Damping Coefficient (γ):**  
   A black numbers and a white background

   Description automatically generated
2. **Calculate Natural Frequency (ω₀):**  
   A square root of a mathematical equation

   Description automatically generated
3. **Calculate Damped Frequency (ω):**  
   
4. **Final Equation of Motion:**  
   

**Amplitude Decay Over Time:**

To assess the decay, I calculated the amplitude after one period :

A black text with numbers

Description automatically generated

The amplitude dropped from 25 cm to 10 cm after one cycle, illustrating significant damping.

**Overdamped Oscillation**

Next, I analyzed the overdamped case by increasing the damping constant. This caused the system to decay without oscillating, following an exponential decay pattern. The equation of motion became:  


Where:

A mathematical equations with numbers

Description automatically generated with medium confidence

: Determined from initial conditions.

**Steps for Derivation:**

1. **Calculate Roots (R₁ and R₂):**  
     
   Substituting values,  
   
2. **Initial Conditions:**

A math equations with numbers and symbols

Description automatically generated

1. **Solve for Coefficients (C₁ and C₂):**  
   Using substitution,  
   A black text on a white background

   Description automatically generated
2. **Final Equation of Motion:**

****

The second term decayed rapidly, leaving the first term to dominate the motion.

**Conclusion**  
Analyzing underdamped and overdamped systems reinforced my understanding of oscillatory behavior and the effect of damping. The equations derived provided a clear mathematical representation of these phenomena, bridging theoretical knowledge with practical applications.

**Underdamped Oscillation**

The motion is computed using the equation

****

The amplitude decays exponentially over time while the cosine term governs the oscillation.

% Parameters

A = 0.25; % Initial amplitude (m)

gamma = 0.833; % Damping coefficient (1/s)

omega = 5.71; % Damped angular frequency (rad/s)

t = linspace(0, 5, 500); % Time array (s)

% Equation of motion

x = A \* exp(-gamma \* t) .\* cos(omega \* t);

% Plotting

figure;

plot(t, x, 'b', 'LineWidth', 1.5);

title('Underdamped Oscillation');

xlabel('Time (s)');

ylabel('Displacement (m)');

grid on;

legend('Underdamped Motion');

**Overdamped Oscillation**

The motion follows , where represent decay rates.

There is no oscillation, and the displacement decays asymmetrically due to the two distinct terms.

% Parameters

C1 = 0.298; % Coefficient from initial conditions

C2 = -0.0483; % Coefficient from initial conditions

R1 = -2.32; % Exponential decay rate 1 (1/s)

R2 = -14.3; % Exponential decay rate 2 (1/s)

t = linspace(0, 5, 500); % Time array (s)

% Equation of motion

x\_overdamped = C1 \* exp(R1 \* t) + C2 \* exp(R2 \* t);

% Plotting

figure;

plot(t, x\_overdamped, 'r', 'LineWidth', 1.5);

title('Overdamped Oscillation');

xlabel('Time (s)');

ylabel('Displacement (m)');

grid on;

legend('Overdamped Motion');

**Combined Plot for Comparison**

The underdamped motion shows oscillations, while the overdamped motion exhibits a smooth decay.

% Plot both motions

figure;

plot(t, x, 'b', 'LineWidth', 1.5); hold on;

plot(t, x\_overdamped, 'r', 'LineWidth', 1.5);

title('Comparison of Underdamped and Overdamped Oscillations');

xlabel('Time (s)');

ylabel('Displacement (m)');

grid on;

legend('Underdamped Motion', 'Overdamped Motion');